APPENDIX D

Flexibility and Robustness:

Response to Demand Uncertainty by Over- and Under- Capacity

D.1 Introduction

Following the cross-disciplinary literature review in Chapter 5 and the conceptual development of ***flexibility*** and ***robustness*** in Chapter 6, this appendix relates these abstract concepts to the familiar example of supply and demand in production and inventory management so as to derive basic cost measures. Such quantifiable costs and their relationships are then applied to other areas to further illustrate the differences between them. Finally, flexibility and robustness are discussed within the context of modelling uncertainty in electricity capacity planning.

Flexibility is the ***ability to react or change***, and robustness is the ***lack of a need to react or change***. Flexibility denotes ***immediate responsiveness*** while robustness provides ***an insurance or cushion against undesirable events***. For example, on a windy day, neither the willow nor the oak tree will collapse because the former bends with the wind (flexibility) and the latter withstands the wind (robustness). Flexibility implies a ***future cost*** as it is a defense against the unexpected, e.g. the cost of producing additional goods to meet a sudden and unexpected rise in demand. Robustness, on the other hand, implies a ***present holding cost*** typically incurred by extra capacity or high inventory levels to meet expected rises in demand.

Although many types of flexibility exist, Gerwin (1993) insists that they are only relevant in response to given types of uncertainty. This paper considers ***demand uncertainty*** only, thus corresponding to Gerwin's ***volume flexibility*** which permits increases or decreases in aggregate production level. Cazalet et al (1978) have looked at the implications of building over and under capacity with respect to demand uncertainty from a decision analysis perspective. Here, we illustrate the costs associated with such production levels to illustrate both flexibility and robustness. Gerwin and others also mention necessary elements in defining flexibility, which we have found earlier in our cross-disciplinary review to include *range* and *time*. *Range* refers to the amount of change, and *time* refers to the length of time to make the change. In this chapter, *range* refers to levels of production that can be assigned or achieved, while *time* is the lead time to produce. Gerwin also observed that the *time* aspect of flexibility has received much less attention than the *range*. For this reason, we will expand on the *time* aspect.

D.2 Simple Example: no lead time, demand = supply, planned = actual levels

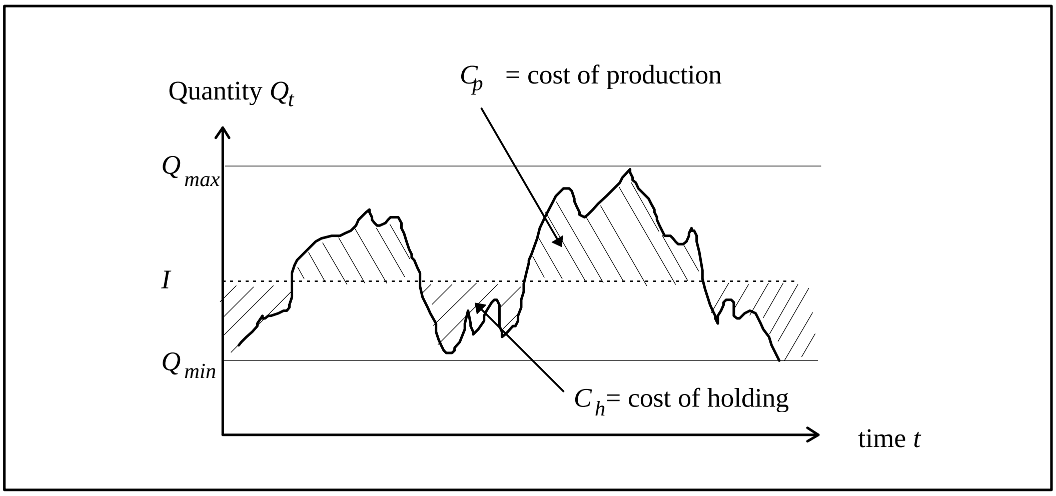
We take a simple case to illustrate the difference between flexibility and robustness. A firm produces the quantity q_t at time t to meet exactly the demand d_t at time t, i.e. $q_t = d_t$. This occurs when lead time is zero and the actual quantity produced is the same as the actual demand at time t. Lead time T is defined as the time it takes to produce the product which is demanded. Q and D represent planned production and forecasted demand, while q and d represent actual levels of production and demand respectively. Capital letters Q and D denote planned whereas small letters q and d denote actual. For the moment we do not distinguish between planned and actual, so that it is only necessary to use Q_t to represent both quantities of production and demand at time t. Let I_t denote the normal production level at time t. This firm chooses to fix I_t at a constant level I. I^{opt} is the optimal level of normal production which minimises the total cost. C_h is the cost of holding the produced quantity when demand is less than I. C_p is the cost of producing quantities above I when demand is greater than I. The following table D.1 lists the basic terminology and notations used in this paper.

Table D.1 Terminology and Notations

variable	planned or expected	actual	planned = actual
supply, production	Q_t	q_t	Q_t
demand	D_t	d_t	D_t
normal production level	$I_t = I$	I	I
minimum , maximum	Q_{min}, Q_{max} D_{min}, D_{max}	q_{min}, q_{max} d_{min}, d_{max}	Q_{min}, Q_{max}
cost of normal production			C_{np}
cost of production (above I)			C_p
cost of holding (below I)			C_h
lead time			T
cost of not meeting demand			C_d
cost of extra production (above Q_{max})			C_{xp}
statistical distribution function for production			$f_t(Q_t)$
average cost of production up to time t with I level of production			$C_t(I)$

In the simple case where production quantity equals demand and planned equals actual, the demand curve is the same as the production curve, as illustrated in figure D.1. The cost of holding and production, C_h and C_p, apply to the areas delineated by the line I and the demand curve. C_p is not the cost of normal production C_{np} which is used to calculate I, but the cost of production beyond normal production to meet demand.

Figure D.1 Costs of holding and production: C_h and C_p



The average cost up to time t with normal output at I is the sum of the cost of normal production C_{np} times the normal production level I, the holding cost C_h times the amount not sold ($I - Q_t$ for $I > Q_t$), and the cost of production C_p times the additional quantity produced ($Q_t - I$ for $I < Q_t$).

$C_t(I) = C_{np} * I$	normal production cost
$+\int_0^t E[C_h(I, Q_t)]\, dt$	holding cost
$+\int_0^t E[C_p(I, Q_t)]\, dt$	production cost

D.2.1 Proportional Cost

Assuming linearity, we have the following relations:

$C_{np}(I, Q_t) = C_{np} * Q_t$	for $I = Q_t$	normal production cost
$C_h(I, Q_t) = C_h(I - Q_t)$	for $I > Q_t$	holding cost
$C_p(I, Q_t) = C_p(Q_t - I)$	for $I < Q_t$	production cost

D.2.2 Flexibility

Flexibility means the ability to change or react when necessary. This can be achieved in several ways. It does not matter what I is as long as the firm has the means to meet demand at whatever level, either by adjusting the level of production as necessary or by producing additional quantity from elsewhere. Alternatively, instead of fixing I_t to a constant level I, the firm can choose a fluctuating $I_t = Q_t$, though this may incur additional cost. Suppose we set I to the minimum demand level, $I = Q_{min.}$

Expected cost is then

$$C_t\ (Q_{min}) = \int_0^t \int_{Q\,\min}^{Q\,\max} C_p\ (Q_t - Q_{min})\ f_t(Q_t) dQ_t dt$$

which depends on $f_t(Q_t)$ and C_p.

D.2.3 Robustness

Robustness means the absence of a need to change or react. When the level of normal production is set at maximum demand Q_{max}, it is not necessary to change the level of production because demand will never exceed Q_{max}. (Recall that we assume maximum demand at Q_{max}).

$I = Q_{max}$

by substitution,

$$C_t\ (Q_{max}) = \int_0^t \int_{Q\,\min}^{Q\,\max} C_h\ (Q_{max} - Q_t)\, f_t\, (Q_t)\ dQ_t\ dt$$

which depends on $f_t(Q_t)$ and C_h.

D.2.4 Flexibility versus Robustness

From these above two equations for C_t, we can tell which is cheaper (hence better). For the risk neutral firm, we conclude the following:

Robustness is better than flexibility if $C_t\,(Q_{max}) < C_t\,(Q_{min})$.

Flexibility is better than robustness if $C_t\,(Q_{max}) > C_t\,(Q_{min})$.

D.2.5 Optimal Policy I^{opt}

The choice of an optimal level of normal production I^{opt} is determined by finding I such that $C_t(I)$ is minimised, as follows.

$$I^{opt} = \underset{I}{Min}\left[\, C_t\,(I) = \int_0^t \left[C_h\,(I - Q_t) \mid \{I > Q_t\} + C_p\,(Q_t - I) \mid \{I < Q_t\}\right] dt \,\right]$$

D.2.6 Special Cases

To find I^{opt}, we consider two special cases of C_h and C_p. If the cost of holding and the cost of production are equal, I^{opt} cannot be uniquely determined. If these costs are not equal, then we can solve for I^{opt} by making a simplifying assumption, that the statistical distribution for Q_t is independent of time.

$C_h \neq C_p$ and $f_t(Q_t) = f(Q)$

Let $f(Q) = \dfrac{1}{Q_{max} - Q_{min}}$ (uniform distribution)

$$C_t\,(I) = \frac{t}{Q_{max} - Q_{min}}\left[\, C_h \int_{Q_{min}}^{I} (I - Q_t)\, dQ_t + C_p \int_{I}^{Q_{max}} (Q_t - I)\, dQ_t \,\right]$$

$$C_t\,(I) = \frac{t}{Q_{max} - Q_{min}}\left[\frac{C_h\,(I - Q_{min})^2}{2} + \frac{C_p\,(I - Q_{max})^2}{2}\right]$$

$$= \frac{t}{2(Q_{max} - Q_{min})}\left[\, C_h\,(I - Q_{min})^2 + C_p\,(I - Q_{max})^2\right]$$

Setting the first derivative to 0, we solve for I^{opt}

$$\frac{dC_t(I)}{dI} = 0 = \frac{t}{Q_{\max} - Q_{\min}}[C_h(I - Q_{\min}) + C_p(I - Q_{\max})]$$

$$C_h (I - Q_{min}) + C_p (I - Q_{max}) = 0$$

$$C_h I - C_h Q_{min} + C_p I - C_p Q_{max} = 0$$

$$I (C_h + C_p) = C_h Q_{min} + C_p Q_{max}$$

Solving for I for I^{opt},

$$I^{opt} = \frac{(C_h Q_{\min} + C_p Q_{\max})}{(C_h + C_p)}$$

If $C_h = 0$ then $I^{opt} = Q_{max}$ (robustness)

If $C_p = 0$ then $I^{opt} = Q_{min}$ (flexibility)

D.2.7 Relative Costs

How do C_h and C_p relate to I^{opt}? Differentiating I^{opt} by C_h, we find a negative relationship between the two.

$$\frac{dI^{opt}}{dC_h} = \frac{Q_{\min}(C_h + C_p) - (C_h Q_{min} + C_p Q_{max})}{(C_h + C_p)^2}$$

$$= \frac{C_p(Q_{\min} - Q_{\max})}{(C_h + C_p)^2} < 0$$

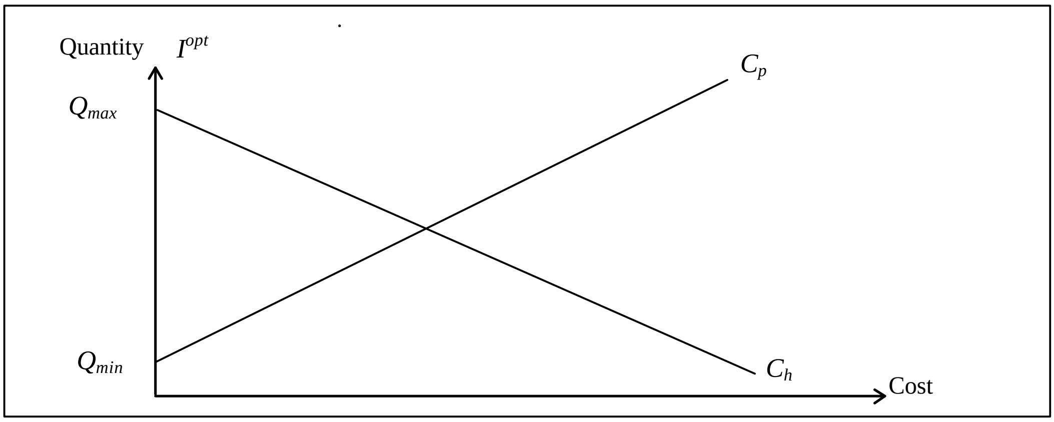
But when we differentiate I^{opt} by C_p, we find

$$\frac{dI^{opt}}{dC_p} = \frac{Q_{\max}(C_h + C_p) - (C_h Q_{\min} + C_p Q_{\max})}{(C_h + C_p)^2}$$

$$= \frac{C_h(Q_{\max} - Q_{\min})}{(C_h + C_p)^2} > 0$$

This implies that as the cost of holding increases, I^{opt} decreases, and we should hold less. Likewise, as the cost of production increases, we should increase I^{opt} or hold more. This agrees with common sense, and we can see it below in figure D.2. These relationships can also be found by evaluating I^{opt} when $C_h > C_p$ and when $C_h < C_p$. When C_h and C_p are unequal and nonzero, I^{opt} will be somewhere between Q_{min} and Q_{max}.

Figure D.2 Relationship between I^{opt} and C_h, C_p



D.3 Extensions of Simple Example

To make the problem more realistic, we vary the basic conditions to examine the effects of I, C_d, T, risk attitude, Q and D, and D and d.

D.3.1 Levels of I

The *range* aspect of flexibility translates into the number of levels of production. Robustness implies fixing I_t at a constant I, whereas flexibility allows changing I_t. Maximum flexibility occurs when normal production level I_t can be set to D_t. Alternatively, robustness also applies to the situation where $Q_{max} > d_{max}$ but flexibility refers to $I_t = Q_t = D_t$.

D.3.2 Cost of Not Meeting Demand C_d

Consider the situation where demand may not be met by existing production capacity. If demand D_t is not met, future demand may fall, because customers can switch to other suppliers. If this firm does not have means to meet demand above D_t, it is inflexible. The cost of extra production beyond maximum capacity C_{xp} can be assessed relative to the cost of not meeting demand C_d and the available means of achieving this.

Suppose D_t is never more than Q_{max}, i.e. P ($D_t > Q_{max}$) = 0. This may happen for the following reasons. In an efficient market, price rises as demand rises. A price rise will deter further increase in demand beyond, say, Q_{max}. However, when prices are fixed or when demand is price-insensitive, D_t may exceed Q_{max}. A second reason could be that when D_t reaches a certain percentage of Q_{max}, it signals the producer to purchase or rent additional machinery to increase total production capacity. Finally, it could well be that the producer has no interest in meeting the D_t that exceeds Q_{max} because it is too costly or impossible. For example, the lead time to acquiring additional production capacity may be too long or the producer has no more physical space to accommodate additional machines. Alternatively, a monopolist without any obligation to meet the additional demand may prefer to ignore it. As long as the cost of not meeting demand is significant and nonzero, the firm needs to consider flexibility. Thus C_d is not only an economic cost but also an opportunity cost and one reflecting the cost of contractual or social obligations.

D.3.3 Effect of Lead Time T

Consider the effect of lead time T, so that the quantity produced is not the same as the quantity demanded, and at best, $Q_t = D_{t+T}$. As long as the cost of not meeting demand C_d is zero, i.e. the firm has no obligation to meet demand, and the lead time to production has no effect on costs. However, if there is a cost to not meeting demand, flexibility and robustness apply. Under these circumstances, if lead time is zero, flexibility is better. If lead time is nonzero, robustness is better. In the case of non-zero lead time and a zero cost of not meeting demand, the firm must decide the benefits and costs of additional revenue. This discussion is summarised in table D.2.

Table D.2 Lead Time and Cost of Not Meeting Demand

	Cost of not meeting demand C_d	
Lead time *T*	$C_d = 0$	$C_d > 0$
$T = 0$	no need	flexibility
$T > 0$	discretion	robustness

D.3.4 Risk Attitude

The above analysis assumes that the decision maker, or the firm producing the goods, is risk neutral. The decision maker's risk attitude affects his preference for flexibility or robustness only when there is a chance that D_t may exceed Q_{max}. If lead time is zero, all else being equal, the decision maker's attitude to risk does not affect his preference for flexibility or robustness. For nonzero lead time and nonzero cost of demand, the risk averse decision maker would prefer robustness to flexibility as the latter implies a risk that demand may not be met on time and that future demand may be affected as a result. Thus, even if the cost of production is lower than the cost of holding, the risk averse decision maker would prefer a higher level of normal production to avoid the risk that the lead time (for extra production) would entail. Implicitly, the cost of holding includes the cost of expired goods if not sold. Table D.3 classifies the decision maker's preferences with respect to his risk attitude.

Table D.3 Preferences with respect to Risk Attitude when P(D_t > Q_{max}) >0

Risk Attitude Situation	risk averse	risk neutral	risk taking
T > 0 and C_d >> 0	robustness; set I > Q_{max}	robustness with flexibility	flexibility

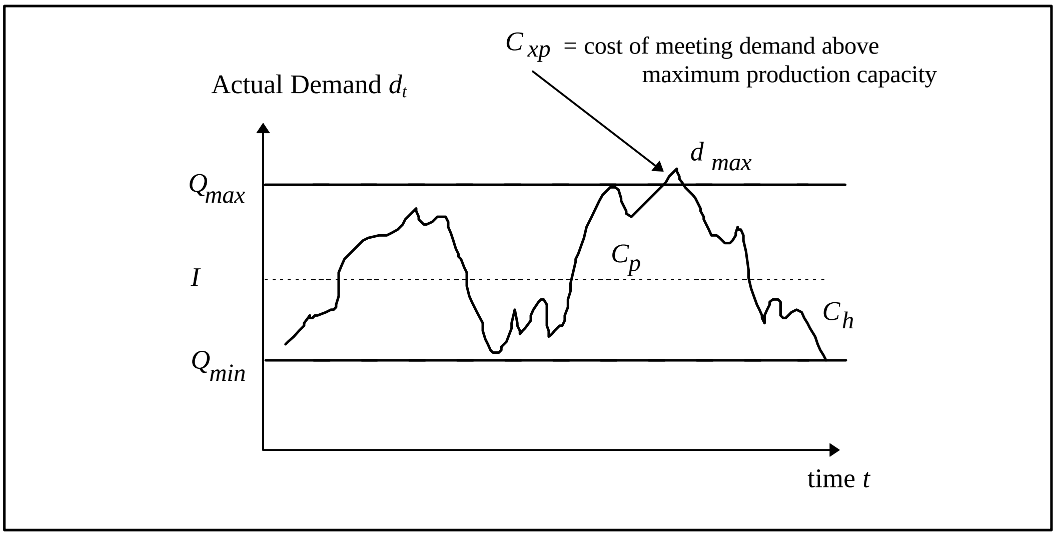
D.3.5 Levels of Q_{min}, Q_{max} with respect to D_{min}, D_{max}

Robustness implies setting Q_{min} and Q_{max} to cover D_{min} and D_{max}. This translates into minimising the probability that D_{max} exceeds Q_{max} and ensuring that Q_{min} can fall to D_{min} without cost. Flexibility, on the other hand, implies fluctuating production levels either by $I_t = Q_t$ or maintaining dynamic Q_{min} and Q_{max} to meet changing demand. Thus D_{max} can be greater than Q_{max} and D_{min} less than Q_{min} provided the capability to meet the discrepancy between D_t and Q_t exists.

D.3.6 Forecasted Demand versus Actual Demand

Suppose that actual demand d_t may exceed expected maximum demand D_{max}. In figure D.3 below, the area between d_{max} and Q_{max} refers to the cost of extra production C_{xp}. This is the average unit cost of producing beyond maximum production capacity.

Figure D.3 Cost of extra production C_{xp}



In this case, d_{max} cannot be predicted with accuracy. No matter what level of I or the level of Q_{max}, there is always a chance that demand will exceed it. If we set $I = Q_{max}$, and d_{max} turns out to be greater than Q_{max}, some demand will not be met. If $d_{max} < D_{max} < Q_{max}$, then we have incurred substantial holding cost, especially between d_{max} and Q_{max}. To determine I^{opt} in this case, we need to consider the probability that d_{max} exceeds Q_{max} or d_t exceeds D_{max}, the cost of not meeting demand C_d, and the cost of extra production beyond given maximum production capacity C_{xp}. Table D.4 summarises the conditions below.

Table D.4 Conditions for Robustness and Flexibility

Cost of not meeting demand:	$C_d > 0$	
Cost of flexibility: C_{xp}, C_p Probability of $d_{max} > Q_{max}$	$C_{xp} = C_p$	$C_{xp} > C_p$
low	robustness	robustness, some flexibility
high	flexibility	high robustness, flexibility

From the above, we conclude that as long as there is a chance that demand may exceed maximum production capacity and the cost of not meeting demand is not zero, some flexibility is necessary. Where cost of production beyond maximum production C_{xp} exceeds cost of production C_p, additional robustness is required. So,

If $P(d_{max} > Q_{max}) > 0$ and $C_d > 0$ then flexibility is necessary.

D.3.7 Errors in Forecasting, Modelling, and Planning

The existence of lead time, holding cost, production cost, extra production cost, and cost of not meeting demand implies a need to forecast and plan ahead. Typically, future demand is forecasted so that D_t can approximate d_t, with the objective of reducing forecasting error, by minimising deviations between d_t and D_t or by ensuring that D_{max} exceeds d_{max}. Production levels are managed so that planned Q_t approximates actual q_t. The more complicated the system, the more likely we can expect errors in forecasting future demand, modelling of the system, and planning decisions to occur. Robustness gives a present known cost of holding by fixing I_t at a level to cover the maximum demand expected. On top of this, flexibility is needed to cover the errors described above.

D.4 Applications by Further Examples

The above analysis can be applied to any situation involving the control of supply and demand. We illustrate the concepts of flexibility and robustness further through two examples. The first example concerns bank customers' preferences for flexibility in their maintenance of the chequebook. The second example concerns a firm's decision to buy or rent machinery.

D.4.1 Example 1: Current and Savings Accounts

A bank customer prefers to keep as low a balance as possible in the non-interest-bearing current account and as high a balance as possible in the interest-bearing savings account. In addition, he would like to reduce the amount of time spent on monitoring his current account. Insufficient balance in the current account means that the cheque will bounce and he will be charged a fee C_d (cost of not meeting demand). The credit balance in the current account represents normal production level I_t, the cost of which is the opportunity cost of not earning interest in the savings account C_h. The cost of transferring between accounts is the cost of production C_p. Lead time T is the amount of advance notice he has to give to the bank or the length of time it takes to transfer between the accounts. The total amount of money he has between the two accounts is Q_{max}. The minimum balance in the current account is Q_{min}. The maximum total withdrawal from the current account is D_{max}. If the customer is flexible (has the capability to be flexible) and prefers it, he would keep I_t (the balance in the current account) as low as possible. If the customer is risk averse and also prefers not to have to monitor or transfer between accounts too frequently, he would keep a high balance in the current account, hence the robustness option. Thus, the cost of robustness is the cost of holding, i.e., opportunity cost of the positive balance in the current account. The cost of flexibility is the extra effort required to monitor and transfer between accounts C_p plus associated transaction or enabling costs.

Some banks are offering all kinds of financial packages to suit customers' preferences. The uncertainties and risks these banks face with regard to customers' frequency and amount of transfers between the accounts are implicitly built into the fees they charge. These fees include non-interest-bearing balance and actual charges. Variations of the above include interesting bearing current account with minimum balance, where the interest is still lower than savings and other types of longer term accounts. There is also a facility for automatic transfer between accounts at the cost of keeping a minimum balance, fixed set up fee, or transaction fee per transfer. Alternatively, negative production levels can be associated with an overdraft facility on the current account.

This analysis may also be applied to money market accounts, off-shore accounts, and financial instruments which give customers the robustness and flexibility required to cope with the demand uncertainty.

D.4.2 Example 2: Buying versus Renting

The buy or rent decision is not only an accounting issue but also a strategic one, affecting the way a firm can deal with future uncertainty. In accounting terms, buying is very different from renting machinery, as one of ownership and control versus borrowing. The former becomes an asset, gets entered into the balance sheet, gets depreciated, and eventually has scrap value. The latter becomes an expense, reduces taxable profit, and does not get carried over to the following year. Strategically speaking, buying ties the firm to this specific technology for the life of the machinery whereas renting enables the firm to switch to new technologies when necessary. Ownership and “borrowing” differ by the degree of commitment or confinement to a specific technology. By renting, the firm can terminate its commitment at any time and limit its technological confinement. Ownership, particularly of a capital asset, pays off if the capital cost discounted over the life of the asset is less than the total cost of renting over the same number of years. In practice, a firm may choose to own most of the machinery and rent additional ones when necessary. This arrangement can be seen as robustness to deal with expected demand and flexibility to deal with the unexpected. Here cost of holding is the purchase cost which reflects the opportunity cost of the machine. Cost of production is the cost of renting. Lead time is translated into how soon the firm can rent or return the extra machine required. The shorter the lead time, the more flexible and attractive is the option of renting.

D.5 The UK Electricity Supply Industry

In the UK electricity industry, the business of generation and the responsibility of meeting demand are no longer borne by one single utility, i.e. the defunct Central Electricity Generation Board CEGB. Q_{max} is the maximum electricity production capacity but not a constant level due to scheduled and unscheduled maintenance and different availabilities of plant. For this reason, normal production level $I_t < Q_{max}$. Furthermore, Q_{max} is the aggregate of plant capacity of the utilities in this deregulated industry and therefore more difficult to determine. Meanwhile as new plants are commissioned and old uneconomic plants retired or sold, Q_{max} will change accordingly. The expression for Q_{max} thus approximates the actual $q_{max,t}$:

$Q_{max} \approx Q_{max,t} = \sum_{i=1}^{n} Q_{max,i,t}$ where $Q_{max,i,t}$ is the maximum capacity of each utility i at time t.

The cost of not meeting electricity demand is very high, especially in this competitive environment. The cost of not meeting demand C_d is translated into reliability stipulations in the contracts between various parties involved. The probability that actual demand exceeds maximum forecasted demand could be positive. Traditional approaches have tried to deal with this uncertainty by first forecasting demand, setting a reserve margin R above expected peak demand based on the volatility of past demand, and finally optimising to produce a minimal costing capacity expansion plan ($Q_{max} = R + D_{max}$). Thus over-capacity is a way to ensure that demand D_t will always be met, but it is not sufficient to guarantee that d_t will always be met. Over-capacity, like under-capacity, is costly. Thus robustness, in the form of over-capacity, is not sufficient.

The cost of robustness C_r is the holding cost of over-capacity. Cost of flexibility can be seen as the cost of responsiveness, immediate change, additional production to meet demand. $C_f = C_{xp}$ + transaction or enabling costs C_x. C_p does not exist as $I = Q_{max}$. Thus we have:

$C_r = C_h$ (robustness)

$C_f = C_{xp} + C_x$ (flexibility)

The cost of flexibility C_f does not include cost of holding which is an ongoing present cost. Instead, C_f includes future transaction and production costs. Thus robustness is a kind of flexibility if the cost of holding can be passed to someone else or if it is zero. A holding cost should also contain an interest or discount rate to reflect the time over the period of holding. Robustness means the need not to change, hence $I_t = I$. Robustness diminishes if I_t is not constant, as there is a cost to changing normal production levels. As I_t tends towards continuous different levels, one reaches flexibility.

How does one gain flexibility? An extremely flexible option has no holding cost and gives instant response. Importing power, for instance, shifts the holding cost (of the plant) to someone else, e.g. Scotland or France. Combined Cycle Gas Turbines (CCGT) are quick to build and require minimal warm-up time. Mandelbaum's (1978) six sources of flexibility suggest different ways to gain flexibility. Demand Side Management (DSM) as practised by many utilities in the USA refers to the use of incentives to lower or shift demand so that D_t will never exceed Q_{max}. Other types of contractual arrangements, such as having a break clause in building a new type of plant, also offer flexibility. Thus flexibility can be translated as having lots of different alternatives (availability and access) to respond to excessive D_t and being able to use them at minimal cost and timing.

Portfolio Theory (Markowitz, 1952) recommends keeping a well-balanced and diversified portfolio in order to maximise returns and spread risks. In electricity generation, diversity in the capacity mix ensures security of fuel supply. Diversity is a double-edged word, containing implications of robustness and flexibility. A utility with a diversified capacity mix, i.e. different types of technologies, different fuels, and different plant lives and other characteristics, is not only protected from fuel supply disruptions (robustness) but also has different alternatives to cope with the unexpected (flexibility). But once again, due to nonzero holding cost, uncertain Q_{max} , and a fuel supply disruption possibility, diversity in the capacity mix gives both robustness and flexibility.

Because flexibility requires the consideration of new solution alternatives with respect to uncertainty, we need to take a modelling approach that facilitates the analysis of choice and uncertainty. This motivates the use of decision analysis as demonstrated in Chapter 8. Scenario analysis is another method to analyse uncertainty. A robust option is one which is good for scenarios 1 to n. By having many options 1 to m that is available for any scenario that arises, but ultimately option j is good for scenario j for instance, then we have flexibility.

D.6 Conclusions

We have examined the differences between flexibility and robustness and the conditions under which they are useful by means of derivation of cost measures. The simple case of production is extended to two familiar examples to illustrate these concepts further. While strict uncertainty makes it difficult to assess flexibility directly, we argue that errors in forecasting and modelling require consideration of flexibility as robustness is not enough.

We have shown that flexibility and robustness are not the same, neither are they opposites. Flexibility is forward looking, reflecting a potential capability, useful when more information can be expected, and implying a future cost. Robustness, on the other hand, is backward looking, minimising regret and a present cost. We can only use it as it is, whereas flexibility offers the potential to change and transform.

We have only looked at demand uncertainty, and at the case of demand being greater than supply. This study can be extended to look at the other side, where demand is lower than minimum supply. This is particularly applicable to the electricity supply industry where over-capacity means increased cost to customers. Research in the application of flexibility and robustness can be extended in many directions, to include responses to other types of uncertainty, increasing complexity in the system, and other areas of application such as regulations, computer systems, and long term investment planning.